

# **WAVEGUIDE HANDBOOK**

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# WAVEGUIDE HANDBOOK

*Edited by*  
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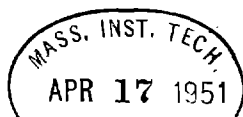
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## *Foreword*

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THE tremendous research and development effort that went into the development of radar and related techniques during World War II resulted not only in hundreds of radar sets for military (and some for possible peacetime) use but also in a great body of information and new techniques in the electronics and high-frequency fields. Because this basic material may be of great value to science and engineering, it seemed most important to publish it as soon as security permitted.

The Radiation Laboratory of MIT, which operated under the supervision of the National Defense Research Committee, undertook the great task of preparing these volumes. The work described herein, however, is the collective result of work done at many laboratories, Army, Navy, university, and industrial, both in this country and in England, Canada, and other Dominions.

The Radiation Laboratory, once its proposals were approved and finances provided by the Office of Scientific Research and Development, chose Louis N. Ridenour as Editor-in-Chief to lead and direct the entire project. An editorial staff was then selected of those best qualified for this type of task. Finally the authors for the various volumes or chapters or sections were chosen from among those experts who were intimately familiar with the various fields and who were able and willing to write the summaries of them. This entire staff agreed to remain at work at MIT for six months or more after the work of the Radiation Laboratory was complete. These volumes stand as a monument to this group.

These volumes serve as a memorial to the unnamed hundreds and thousands of scientists, engineers, and others who actually carried on the research, development, and engineering work the results of which are herein described. There were so many involved in this work and they worked so closely together even though often in widely separated laboratories that it is impossible to name or even to know those who contributed to a particular idea or development. Only certain ones who wrote reports or articles have even been mentioned. But to all those who contributed in any way to this great cooperative development enterprise, both in this country and in England, these volumes are dedicated.

L. A. DuBRIDGE



## *Preface*

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THIS book endeavors to present the salient features in the reformulation of microwave field problems as microwave network problems. The problems treated are the class of electromagnetic "boundary value" or "diffraction" problems descriptive of the scattering properties of discontinuities in waveguides. Their reformulation as network problems permits such properties to be calculated in a conventional network manner from equivalent microwave networks composed of transmission lines and lumped constant circuits. A knowledge of the values of the equivalent network parameters is a necessary prerequisite to quantitative calculations. The theoretical evaluation of microwave network parameters entails in general the solution of three-dimensional boundary-value problems and hence belongs properly in the domain of electromagnetic field theory. In contrast, the network calculations of power distribution, frequency response, resonance properties, etc., characteristic of the "far-field" behavior in microwave structures, involve mostly algebraic problems and hence may be said to belong in the domain of microwave network theory. The independence of the roles played by microwave field and network theories is to be emphasized; it has a counterpart in conventional low-frequency electrical theory and accounts in no small measure for the far-reaching development of the network point of view both at microwave and low frequencies.

In the years 1942 to 1946 a rather intensive and systematic exploitation of both the field and network aspects of microwave problems was carried out at the Radiation Laboratory of MIT by a group of workers among whom J. Schwinger played a dominant role. By means of an integral-equation formulation of field problems, Schwinger pointed the way both in the setting up and solving of a wide variety of microwave problems. These developments resulted in a rigorous and general theory of microwave structures in which conventional low-frequency electrical theory appeared as a special case. As is to be expected, the presentation of the results of these developments involves the work of many individuals both in this country and abroad, as well as much material which is now more or less standard in mathematical and engineering literature. Unfortunately, it has not been possible to document adequately these sources in the present edition. It is hoped that these and

other omissions will be remedied in a subsequent and more up-to-date edition.

Although the primary aim of this book is to present the equivalent-circuit parameters for a large number of microwave structures, a brief but coherent account of the fundamental concepts necessary for their proper utilization is included. Thus there is summarized in the first three chapters both the field and network theoretic considerations necessary for the derivation and utilization of the basic transmission line—equivalent-circuit formalism. The mode concept and transmission-line formulation of the field equations are introduced in Chapter 1. This chapter contains an engineering treatment of the transmission-line theory necessary for the description of propagating and nonpropagating modes in the more important types of uniform and nonuniform waveguides. The field-structure, propagation, attenuation, etc., characteristics of the transmission-line modes so described are compiled in Chapter 2, with both quantitative and pictorial detail. The elements of microwave-network theory required for the analysis, representation, and measurement of the equivalent circuits for  $N$ -terminal microwave structures are outlined in Chapter 3; also contained in this chapter is a sketch of some of the field theoretic methods employed in the derivation of the equivalent-circuit parameters reported in Chapters 4 to 8. Although most of the above material is written for the impedance-minded microwave engineer, some of the sections should be of interest to the applied mathematician.

The remaining chapters contain a compilation of the equivalent-circuit parameters for a variety of nondissipative  $N$ -terminal microwave structures. These results are presented usually both analytically and graphically in individual sections having an intentionally concise format to avoid repetition. Since the analytical formulae are frequently cumbersome to evaluate, care has been taken to achieve a reasonable degree of accuracy in the graphical plots. In Chapter 4 a number of two-terminal structures, such as beyond-cutoff and radiative waveguide terminations, are treated. Obstacle and aperture discontinuities in waveguides, gratings in free space, etc., are among the four-terminal structures described in Chapter 5. Chapter 6 deals with six-terminal microwave structures and contains the equivalent-circuit parameters for a number of  $E$ - and  $H$ -plane  $T$ - and  $Y$ -junctions, bifurcations, etc. Several eight-terminal structures are treated in Chapter 7. Chapter 8 contains the circuit description of a number of typical composite microwave structures: dielectric-filled guides, thick apertures, etc. In contrast to the relatively complicated field calculations employed to obtain the previous results, only simple microwave network calculations are required to find the circuit parameters and properties of these composite structures.



The equivalent-circuit results in the various sections of Chapters 4 to 8 involve the expenditure of considerable time and effort on the part of many workers—often not at all commensurate with the space devoted to the presentation of these results. Each section usually represents the contributions of many individuals who unfortunately are not acknowledged in each instance. In addition to J. Schwinger, the following helped with direct theoretical contributions to these sections:

J. F. Carlson, A. E. Heins, H. A. Levine  
P. M. Marcus, and D. S. Saxon.

Indirect contributions were made by H. A. Bethe, N. H. Frank, and R. M. Whitmer. The efforts of Levine and Marcus, who remained with the office of publications until its close in 1946, are particularly acknowledged; the latter correlated all the tabulated work reported in the appendix. The continued interest and criticism of Levine and Schwinger since the close of the laboratory are greatly appreciated. Although a great deal of experimental work on the measurement of equivalent-circuit parameters was carried out, only that part which is not covered by or in agreement with theory is included in Chapters 4 to 8. The work of W. H. Pickering *et al.*, of California Institute of Technology, and of C. G. and D. D. Montgomery should be cited in this connection.

A considerable amount of technical assistance was rendered by many others. Mrs. A. Marcus did most of the work on the mode plots presented in Chapter 2. C. W. Zabel correlated some of the theoretical and experimental data on  $T$  sections in Chapter 6. Most of the numerical computations were carried out under the direction of A. E. Heins by M. Karakashian, R. Krock, D. Perkins, B. Siegle, and others. Finally, the valuable editorial assistance, planning, and criticism of H. M. James in the initial stages of preparation of this book should be mentioned.

Although conceived at the Radiation Laboratory of MIT, the greater part of this book was written in the years subsequent to its close while the author was a staff member of the Polytechnic Institute of Brooklyn. The author wishes to thank Professor E. Weber, Director of the Microwave Research Institute at the Polytechnic Institute, for use of the technical and clerical facilities of the laboratory in the preparation of this book; also various members of the Institute for their criticism and proofreading of many sections of this book; and lastly his wife, Muriel, for her continuous help and encouragement.

N. MARCUVITZ

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