

**COMPUTING MECHANISMS  
AND LINKAGES**

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# COMPUTING MECHANISMS AND LINKAGES

*By* ANTONÍN SVOBODA

*Edited by* HUBERT M. JAMES

OFFICE OF SCIENTIFIC RESEARCH AND DEVELOPMENT  
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## *Foreword*

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THE tremendous research and development effort that went into the development of radar and related techniques during World War II resulted not only in hundreds of radar sets for military (and some for possible peacetime) use but also in a great body of information and new techniques in the electronics and high-frequency fields. Because this basic material may be of great value to science and engineering, it seemed most important to publish it as soon as security permitted.

The Radiation Laboratory of MIT, which operated under the supervision of the National Defense Research Committee, undertook the great task of preparing these volumes. The work described herein, however, is the collective result of work done at many laboratories, Army, Navy, university, and industrial, both in this country and in England, Canada, and other Dominions.

The Radiation Laboratory, once its proposals were approved and finances provided by the Office of Scientific Research and Development, chose Louis N. Ridenour as Editor-in-Chief to lead and direct the entire project. An editorial staff was then selected of those best qualified for this type of task. Finally the authors for the various volumes or chapters or sections were chosen from among those experts who were intimately familiar with the various fields, and who were able and willing to write the summaries of them. This entire staff agreed to remain at work at MIT for six months or more after the work of the Radiation Laboratory was complete. These volumes stand as a monument to this group.

These volumes serve as a memorial to the unnamed hundreds and thousands of other scientists, engineers, and others who actually carried on the research, development, and engineering work the results of which are herein described. There were so many involved in this work and they worked so closely together even though often in widely separated laboratories that it is impossible to name or even to know those who contributed to a particular idea or development. Only certain ones who wrote reports or articles have even been mentioned. But to all those who contributed in any way to this great cooperative development enterprise, both in this country and in England, these volumes are dedicated.

L. A. DuBRIDGE.

Brooklyn, Dec. 1941

JUL 6 1964



## *Preface*

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THE work on linkage computers described in this volume was carried out under the pressure of war. War gives little opportunity for the advancement of abstract knowledge; all efforts must be concentrated on meeting immediate needs. In developing techniques for the design of linkage computers, the author has therefore been forced to concentrate on finding practical methods for the design of computers rather than on developing a unified and systematic analysis of the subject. The war has thus given to this work a special character that it might not otherwise have had.

The impulse to the development of the methods presented in this volume for the mathematical design of linkage computers grew out of a collaboration of the author with his friend, Dr. Vladimir Vand. That collaboration was begun in France in 1940, and was brought to a premature end by the progress of the war. Though these ideas and methods have largely been developed by the author since that time, he wishes to emphasize that credit for the initiation of the work is shared by Dr. Vand. It must be mentioned also that the techniques described in this book were for the most part developed before the author became associated with the Radiation Laboratory.

The author wishes to express sincere gratitude to Dr. H. M. James, the editor of this volume, who gave the book its present form, contributing many examples and many improvements to the methods. (Secs.: 6-7, 6-8, 6-15, 8-6.)

The book would never have been completed in such a short time without the assistance of Miss Constance D. Boyd, who read the manuscripts, and Miss Elizabeth J. Campbell, Mrs. Kathryn G. Fowler, Miss Virginia Driscoll, and Miss Patrica J. Boland, who calculated the tables and drew nomograms. The author also wishes to thank Dr. I. Maddaus, Jr., for bibliographical research.

The publishers have agreed that ten years after the date on which each volume in this series is issued, the copyright thereon shall be relinquished, and the work shall become part of the public domain.

A. SVOBODA.

PRAHA, CZECHOSLOVAKIA,  
June, 1946.





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